3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Mathematics

Course : MATH1051 (SEC)

(Graph Theory)

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) How many vertices are there in a 4-regular graph with 10 edges?

(b) If G is a graph of order p and size q and has t vertices of degree m and all other vertices of degree n, then show that $(m-n)t + pn = 2q$.

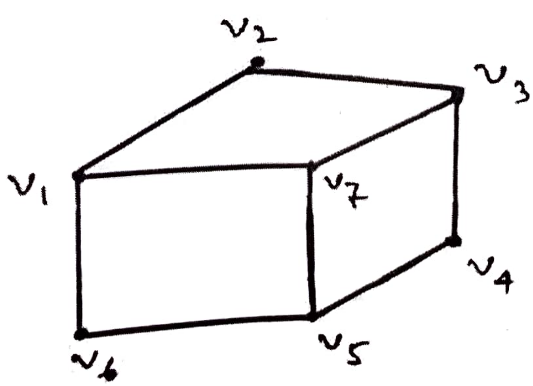
(c) Let G be a connected graph with 8 vertices and 7 edges. Does G contain a vertex of degree 1? Justify your answer.

(d) Show that for any complete graph k_p with p vertices and any vertex v of k_p, $k_p - v = k_{p-1}$ holds.

(e) Let G be a simple connected planar graph with 13 vertices and 19 edges. Then find the number of faces in the planar embedding of the graph.

(f) Define an Eulerian graph and give necessary condition for a graph to be Eulerian.

(g) Examine whether the following simple graph is bipartite or not. Give reason.



(h) Does there exist a tree of order 15 such that the sum of the degrees of the vertices is 30? Justify.

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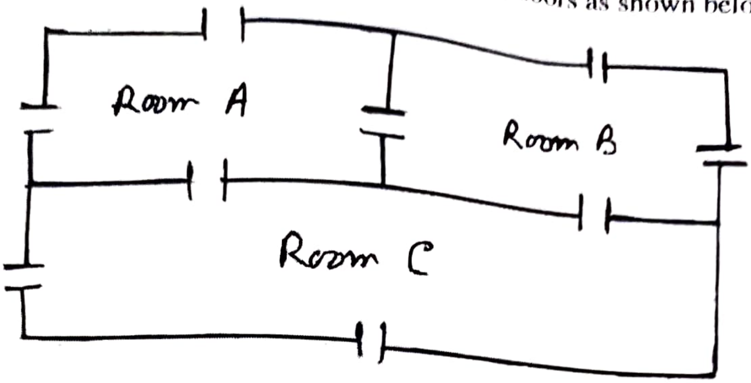
2. Answer *any two* questions: 5×2=10

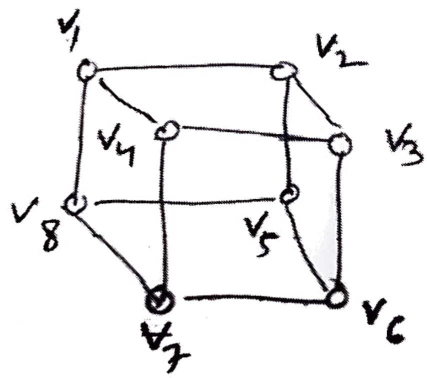
(a) Consider the floor plan of the three-room flat with doors as shown below:

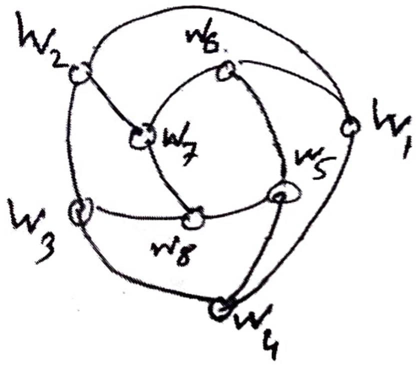


Each room is connected by doors with every other room and outside as shown in the plan. Is it possible to start in a room or outside and take a walk that goes through each door exactly once? If there is such a walk, find it. Can you return to the starting point through this walk? Justify.

4+1

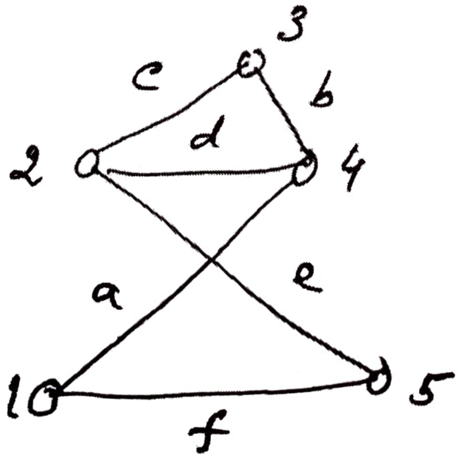
(b) When are two graphs said to be isomorphic? Show that the following two graphs G_1 and G_2 are isomorphic:





1+4

(c) (i) Represent the following graph with an incidence matrix:

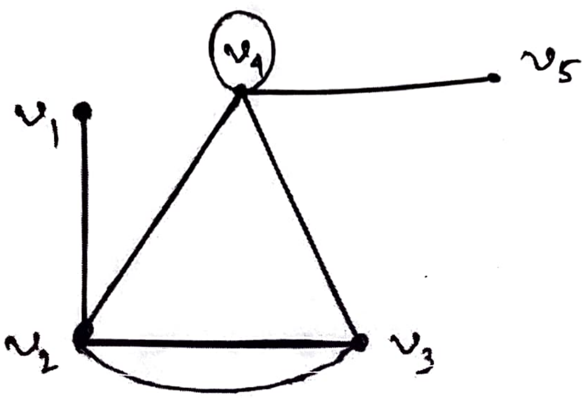


(ii) Draw the digraph G corresponding to the adjacency matrix:

$$A(G) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

3+2

(d) The pictorial representation of a graph G is given below:



G

(i) Find the degree sequence of the graph G.

(ii) Find the pendant vertices in G, if exists.

(iii) Is the graph G regular?

2+2+1

10×2=20

3. Answer *any two* questions:

(a) (i) Prove that a graph with n vertices is a tree if and only if it is acyclic and contains exactly $(n-1)$ edges.

(ii) State and prove Euler's theorem on planar graphs.

4+6

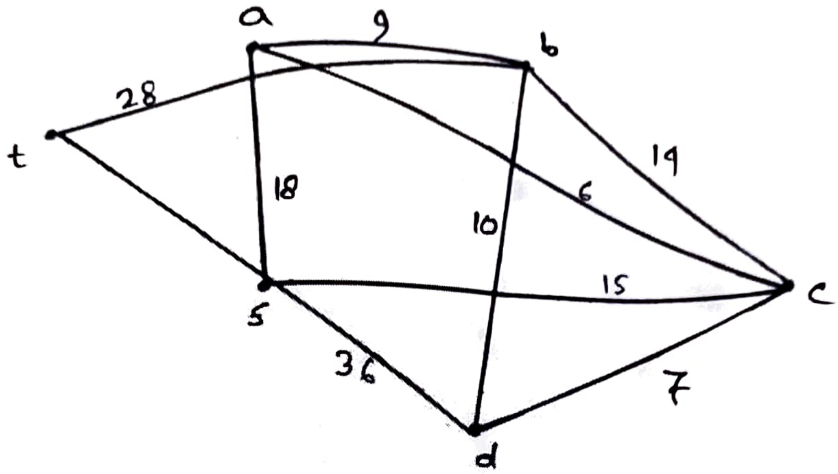
(b) (i) Find an optional Hamiltonian cycle in a directed network whose weight matrix A is given below:

$$A = \begin{pmatrix} - & 17 & 10 & 15 & 17 \\ 18 & - & 6 & 10 & 20 \\ 12 & 5 & - & 14 & 19 \\ 12 & 11 & 15 & - & 7 \\ 16 & 31 & 18 & 6 & - \end{pmatrix}$$

(ii) Prove that any self-complementary graph has $4n$ or $4n + 1$ vertices.

6+4

(c) (i) Applying Dijkstra's algorithm, find a shortest path between the vertices s and t in the following weighted graph G.



G

(ii) Find the smallest positive integer n such that the complete graph K_n has at least 400 edges.

(iii) Decide whether the degree sequence $\{7, 6, 4, 3, 3, 3, 2, 1\}$ is graphical or not. 5+3+2

(d) (i) In a group of nine persons, is it possible for each person to shake hands with exactly five other persons? Justify.

(ii) Draw a graph which is Eulerian but not Hamiltonian. Justify your answer.

(iii) Discuss the concept of graph colouring and its applications. 3+3+4
